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Sine Spiral Graphing

A new method of graphing motion called "Sine Spiral Graphing" was developed by me when I was 16. It allows for simultaneously graphing the sine and cosine curves of an object in motion, three-dimensionally. Sine and cosine, when graphed simultaneously in two dimensions, look like two staggered intersecting waves traveling in the same general direction. (Fig. 1) There has been a need for developing better methods of graphing an object's two-dimensional (flat) motion through space over a period of time that more clearly shows the progression of travel. At present, mapping three-dimensional motion using different variables is more complicated, but could be a further application of the principles presented in the "Sine Spiral Graphing" method. The "Sine Spiral" is based on the spiral shape of two-dimensional circular motion graphed in three dimensions using this new graphing technique. The name is derived from the general name of the sine wave combined with what the actual 3D graph looks like: a spiral. This technique could be helpful for scientists and students alike in many applications. Some possible application for the Sine Spiral could be:

- Plotting the motion of a bead in a hula hoop as it spins around one's waist.
- Calculating the position of various atomic/subatomic particles moving in relation to each other over time.
- Plotting the velocity and position of a point on an automobile wheel as sit spins down a runway or curvy hilly road.
- Plotting the motion of a baseball spinning through the air as it travels forward to the catcher over a period of time.
- Calculating the motion of a point on a bowling ball as it rolls down the lane over time.
- Calculating the speeds and positions of a set of points, on various gears at work, in a clock in relation to each other over time.
- Calculating the motion of a point on a rocket ship, or of a point on a space satellite as it orbits a planet.
- Plotting the movement of a chicken in a tornado.

All of these examples listed present graphing difficulties when depicted on a normal graph. The motions in these examples could be calculated on a computer and represented in a simulated fashion to show the actual movement in space for one point in time at a time. Concurrent Sine Spiral graphs can also be drawn for comparison of points on multiple moving objects. However, it would be difficult to graphically represent these motions for all points in time all at once. A simulation could be like a video, where one can only view one place on the video at a time. Viewing forward and reverse at the same time is not logistically possible on a video. However, when motion is three-dimensionally graphed on a computer using a Sine Spiral, it is possible to view these motions for all points in time all at once. A very effective way to manipulate and browse three-dimensional graphs (such as a Sine Spiral) on a computer is with Virtual Reality equipment. With Virtual Reality equipment, the perspective of the viewer can freely move around in space (on the graph) and see the 3D objects in one's graph from any perspective. In a Virtual Reality graph, the user can have total control over what is viewed and how it is viewed.

Understanding the trigonometric functions of sine, cosine, tangent, and their inverse counterparts is a necessity for understanding Sine Spiral Graphing. Trigonometric functions of real numbers, called "Circular Functions" (or Wrapping Functions), can be defined in terms of the coordinates of points on the unit circle with the equation x^2+y^2=1 having its center at the origin and a radius of 1. (Fig. 2)

There are three elements in a two-dimensional trigonometric function: the angle of rotation (sigma), the radius of the rotation r, and the (x,y) position of the point at that angle and radius. As can be seen in Figure 3, the x and y portions of the graph are always perpendicular to each other. Thus a right triangle is formed between the x, y, and radius sides. Right triangle rules can therefore be applied to this point in space (Brown/Robbins 190).

Such trigonometric functions as sine and cosine can be applied to the triangle formed by rotation. These functions, sin and cos, are of fundamental importance in all branches of mathematics. One can use points other than those on the unit circle to find values of the sine and cosine functions. (Fig. 4) If a point Q has coordinates (x,y), and it is at angle sigma in reference to the origin, (cos sigma) = x/r and (sin sigma) = y/r. To obtain a rough sketch of a sine wave, plot the points (t, sin t) (Fig. 5), then draw a smooth curve through them, and extend the configuration to the right and left in periodic fashion. This gives the portion of the graph shown in Figure 5 (Swolowski 78).

A cosine can be graphed in the same fashion by simply shifting the graph 90 degrees to the right. (Fig. 6) An object's circular motion can be described by either a sine wave or a cosine graphed in the same fashion. Such a wave is composed of the object's radius of rotation and the period (number of degrees in on cycle) per unit of time that it rotates. Seeing an object's sine and cosine graph simultaneously greatly helps in visualizing the object's motion analytically compared how it found in real life. Watching an animation of an object spinning is the same as seeing the x and y coordinates (cosine and sine) of the object for each frame of the animation, one frame at a time. This is because one could see a scale view of its whole two-dimensional motion over a period of time. Visualizing an object's true motion in nature from merely looking at a graph of its sine or cosine can be difficult to conceptualize. For this reason, the Sine Spiral may be an improvement in current co-linear graphing (Fig. 7).

Velocity over the period of one rotation on a sine curve can be measured by dividing the distance traveled in one rotation by the amount of time it takes to complete that one rotation. Velocity = change in distance/change in time + direction.

Any change in velocity (a change in time) will change the distance between peaks of the spiral. The whole Z-axis around which the spiral revolves represents time passed. When the velocity is constant, the distance from peak to peak in the spiral is constant or each distance from one peak to another peak is the same. (Fig. 6) Therefore, if the distance from one peak to another changes somewhere in the spiral, this indicates that the velocity has changed at that point in time.

Within the Sine Spiral, some of the variables that can change in the object's motion are velocity, radius of rotation, position of axis of revolution, and the scale upon which measurements are based. The shape of this spiral is an indication of any and all of these variables. The change in the shape of the spiral correlates to the change in one or more of these variables. (Fig. 7)

Webster's Third New International Dictionary defines a spiral as "A three-dimensional curve (as a helix) with one or more turns around and axis." In current circular motion, the sine of the angle of rotation provides a Y value (Sine=Y/Radius of Rotation), while the cosine of that same angle provides and X value (Cosine=X/Radius of Rotation). These X and Y values are all that is needed to draw the two-dimensional models of rotation known as the sine curve and the two-dimensional models of rotation known as the sine curve and cosine curve (or sine wave). To my knowledge it has not been thought possible to graph this same motion in three dimensions though, because one needs an X, Y, and Z coordinate in order to graph in 3D. There can be an X and Y coordinate by finding the sine and cosine of a unit circle. All that is needed is a Z coordinate to make the circular motion graphable in three dimensions.

That Z coordinate could be representable by time, or speed of rotation, or even the period of degrees it takes for one complete rotation. In a sine wave, the period is 360 degrees. Using the period of degrees in one rotation, one can find a constantly increasing Z coordinate by dividing the current number of degrees traveled by the period of degrees it takes to complete one rotation. In short, degrees/period. The period can be depicted by a set amount of time. Finding a ratio between something that can be used as a reference point (one second v.s. the number of degrees in one rotation) to one's current progress in that measurement scale (number of seconds that have passed v.s. number of degrees that have been traveled) determine where one is on the Z-axis.

By dividing one's progress by a predetermined scale of reference, a new dimension can be generated in which to plot on a graph in order to illustrate this in three-dimensional fashion. This new dimension can be called the "Z-axis". Now that there is an X, Y, and Z dimension available, a three-dimensional model of an object's progress through its path of circular motion is possible.

For 3D motion, one can draw three spirals over the same T axis and where two of the spirals intersect, plot a point. Connecting the dots between the points gives one a tri-spiral (a spiral or shape that represents 3D motion over time). One can continue plotting the points with several objects and where the tri-spirals intersect, the objects intersect. One can break down the tri-spiral to find out where the X, Y, and Z coordinates are in space and the time coordinates of the intersection.

To use the Sine Spiral to map the 3D motion throughout time, one could mark the spiral with tags (or color code it) that tell one when and how far down the Z-axis it travels. Then to graph several objects to compare their motions and positions to each other, one can have a computer draw lines of the same color of the Z-tag, linking all of the objects that intersect on the two planes like the ZX plane, or the ZY plane. That way, one could identify when objects like planets line up on a plane or intersect.

There is much to benefit from in being able to graph an object's progress at the same time as its position in space. One can see time from an outside perspective and also see how an object's motion, position, and speed relate to any point in time. In many circumstances, it may be very useful to finally be able to get to see the general shape of an object's travel through all points in time all at once. This new method of graphing circular motion in three dimensions is the "Sine Spiral".

The graph forms a regularly spaced spiral whose axis is a straight line equidistant from the perimeter of the spiral. Changing the radius of rotation around a center axis changes the radius of the spiral around the Z axis. Changing the center of rotation in two-dimensional space (X, Y coordinates) makes the Z axis of the sign spiral curve up, down, or to the sides when graphed (instead of the normal straight line Z-axis).

For instance, an air hockey puck pinning in place would have a regular sign spiral that represents a point on the puck's perimeter that is traveling in a circle. Now if the spinning puck were to be slid across an air hockey table, that same point (on the perimeter of the puck) would have an irregular sine spiral whose radius would be constant, but the Z-axis around which the graph spirals would instantaneously bend at a ninety degree angle.

A computer can easily generate this three-dimensional picture of an object "N" at point "T" in time if the speed of travel is irregular (or at the ratio of degrees traveled to the period of one complete rotation if the speed is constant). (Fig. 8)

Graphing any two-dimensional motion (motion that moves in any direction on a flat plane), or rotation in three-dimensions using time or progress as the third dimension allows one to look at time from an outside perspective. The Sine Spiral can be used to graph any such two-dimensional motion, or any number of combinations of such motion. It can be used to graph several objects moving around in 2D (flat) space on the same plane. The Sine Spiral can be used to graph an object which has a rotation within a rotation, and so on (Fig. 9). In this case, each next level of rotation is on an incrementally larger scale. To view some of the higher levels of rotation, one must graph the object's motion over a longer period of time. This concept can relate to complex motions of a longer period of time. This concept can relate to complex motions of a large scale found in, for example, the universe. Sine Spiral graphing can literally be used to graph the motion of every particle in perceivable universe for all points in observable time, simultaneously (by bending the Z-axis appropriately to accommodate changes is axis orientation). Using the Sine Spiral, graphing motion in the Z-axis, or time, requires one to employ a means to mark or reference the Sine Spiral in order to distinguish how deep down the Z-axis the motion has traveled.

Without a Sine Spiral, one can only pick three-dimensions to see on a graph for all points in those dimensions. One could have X, Y, and Z coordinates on a 3D graph all at once, but only for one point in time per graph. Or one could illustrate motion in any two dimensions for all points in time using the Sine Spiral. Here are some of the dimensions from which one can choose: X, Y, Z, and Time. One can have four or more dimensions on a graph by selecting 3 variables form out of the X, Y, Z, and Time, as well as any number of descriptive, qualitative, categorical, computational, or other quantitative dimensions. These kinds of dimensions may appeal/apply to one's senses and could be described in "real" dimensions such as the Z-axis and others.

With 3D applications using this concept (once improved methods of graphing 3D motion with the sine spirals are better developed), other more complex spirals can be mapped. Such 3D applications could include the universe in their motion through space throughout all time to see where certain ones meet or line up), and graphing the motion of particles of a sun during a supernova (the spiral would look similar to a tangent spiral as described below). The Sine Spiral may be an improvement in the graphing of nonlinear and linear motion. With the help of the recent Virtual Reality technology, most any computer can be used to build 3D models such as Sine Spirals. We can construct and view a Sine Spiral and have complete control over the graph, viewing it in 3D space as if it were physically here.

There are many new math applications and theorems that may apply to this concept. Different types of spirals are possible with the general Sine Spiral method. Such shapes could include the Sine Tube (a sine spiral whose period is infinitely small), the Tangent Spiral (which uses a sine spiral whose period is infinitely small), the Tangent Spiral (uses the equation Tan sigma = (y/r)/(x/r) for the x and y coordinates), and the secant spiral (uses sec sigma = 1/(x/r) for the coordinate and csc sigma = 1/(y/r) for the y coordinate). Also, in either two-dimensional or three-dimensional motion (when a graphing method is available), an object can be spinning in a circle within a circle (each level of rotation incrementally bigger than the previous), and this will make a very special type of Sine Spiral that looks like a spiral within a spiral within a spiral, etc., depending on how many levels of rotation are going on. More new math applications are sure to be found that can apply to the Sine Spiral as it is used.

Graphing three-dimensional motion with the Sine Spiral is more difficult to do, but can be done effectively. Graphing three-dimensional motion using the Sine Spiral needs further refinement at this time, but will hopefully be available for use in the near future. There are many new avenues that open up as people figure things out in science and math. The Sine Spiral may be another door in mathematics ready to be opened up and entered. Through this door may be a whole new way to look at things, a way to see objects in nonlinear motion from a standpoint outside of time.

Works Cited:

Brown R., and D. Robbins, "Advanced Mathematics: A Precalculus Course"
Boston: Houghton Mifflin Company 1987.

Fleenor C., M. Shanks, and C. Brumfiel. "The Elementary Functions".
Boston: Addison-Wesley Publishing Company, 1973.

Gove, P.B., ed. "Webster's Third New International Dictionary, Unabridged".
Springfield, MA: Miriam-Webster, 1986.

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Manougian, M.N. "Trigonometry with Applications".
Tampa, FL: Mariner Publishing Co., 1980.

Swokowski, E.W. "Fundamentals of Trigonometry".
Boston: Prindle, Weber & Schmidt, Incorporated, 1982.

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Sin θ and Cos θ graphed simultaneously in Two dimensions

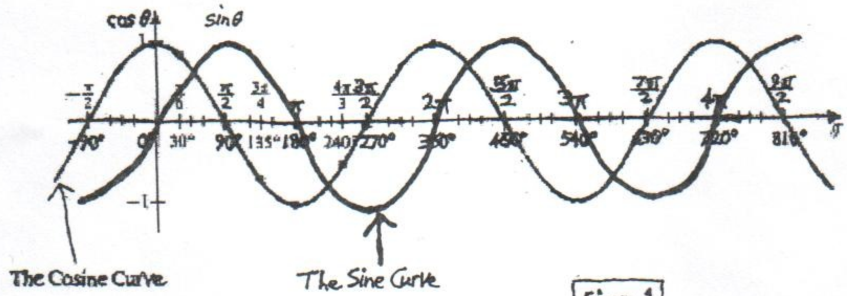


Figure 1

We will define the trigonometric functions of real numbers, called circular functions (or wrapping functions), in terms of the coordinates of points on the unit circle. We consider the unit circle with equation

$$x^2+y^2=1$$

having its center at the origin and a radius of 1 (see Figure 1).

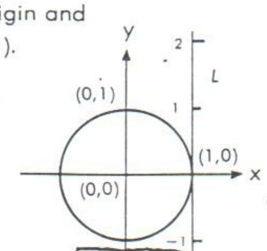


Figure 2

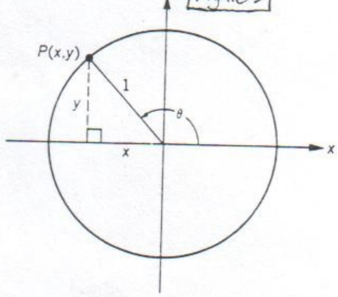
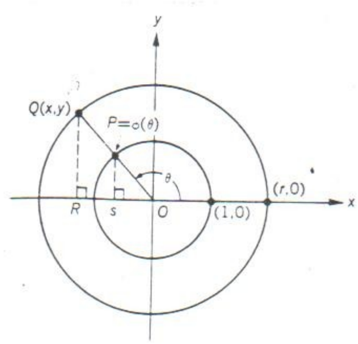


Figure 3



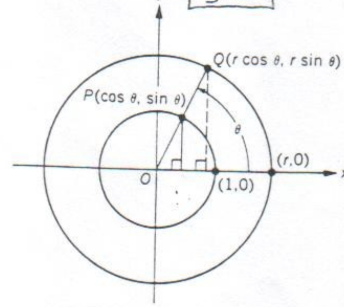


Figure 4

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin t$	0	$\frac{\sqrt{2}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$\frac{-\sqrt{2}}{2}$	-1	$\frac{-\sqrt{2}}{2}$	0

To obtain a rough sketch we may plot the points $(t, \sin t)$ listed above, draw a smooth curve through them, and extend the configuration to the right and left in periodic fashion. This gives us the portion of the graph shown in Figure 2.27. Of

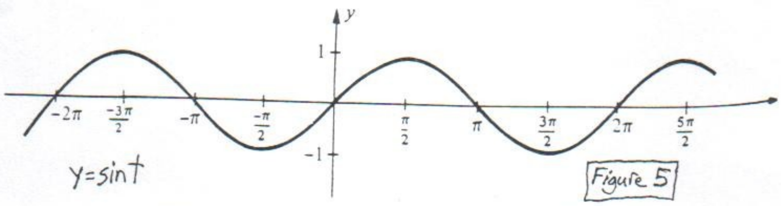


Figure 5

Note that the graph of $y = \cos t$ can be obtained by shifting the graph of $y = \sin t$ to the left a distance $\pi/2$.

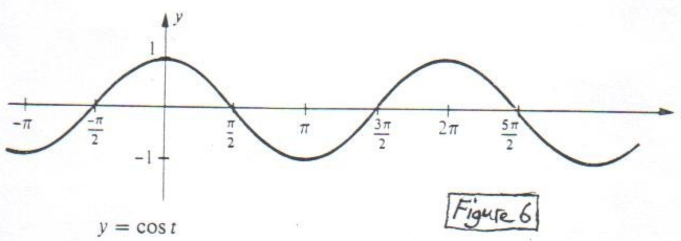
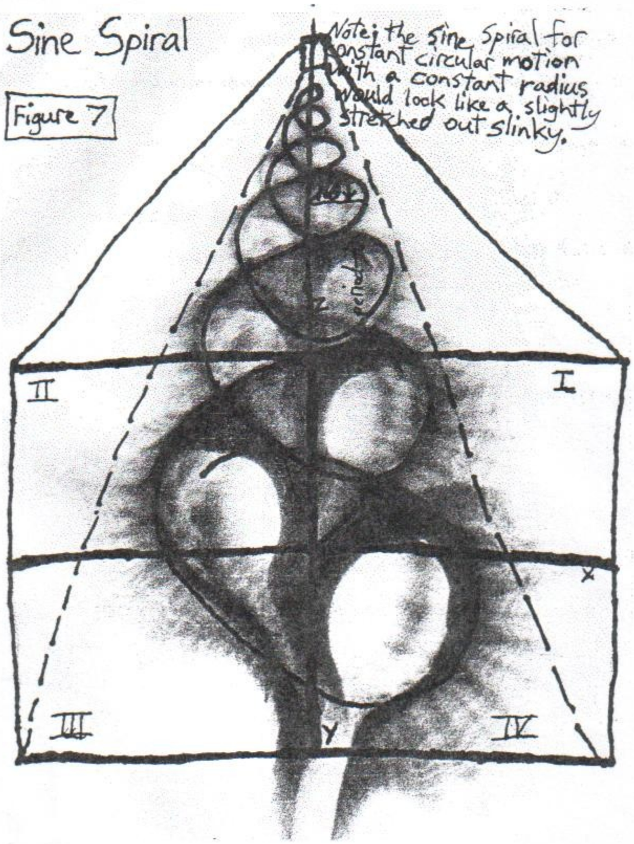


Figure 6

Sine Spiral

Figure 7



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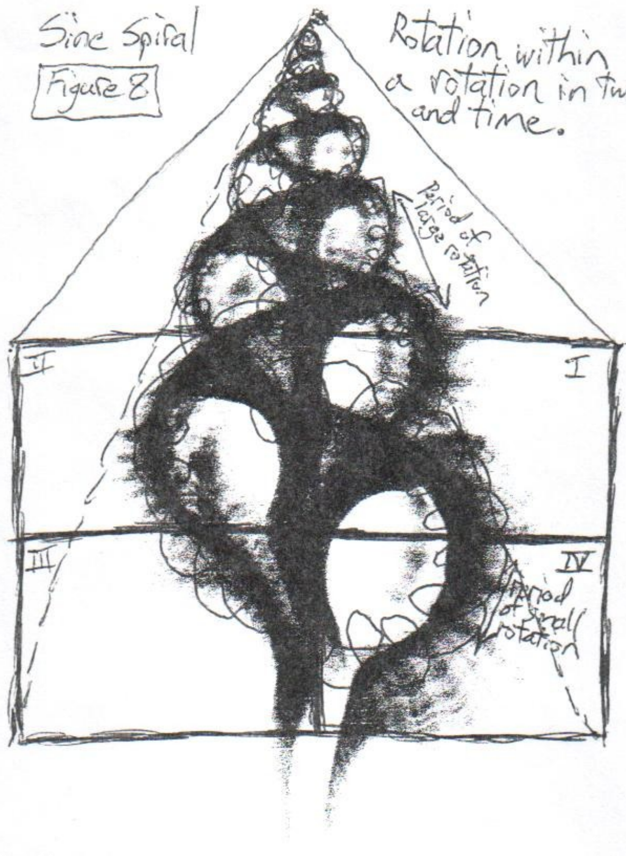
Sine Spiral

Figure 8

Rotation within a rotation in two-dimensions and time.



*special thanks to Joshua Tepper in 2005 at Carnegie Mellon University for helping me program my sine spiral Graphing algorithms into Steven Wolfram's Mathematica visualization software to produce these next four pages of visuals.

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```
In[2]:= Plot[Sin[x], {x, 0, 2 Pi}];
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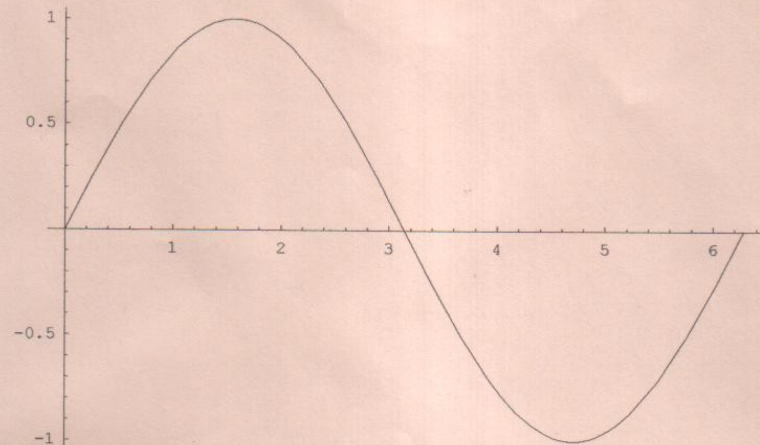
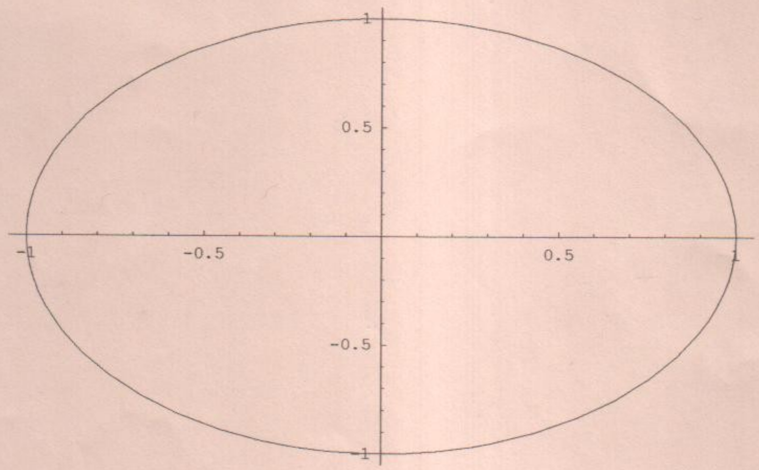



```
In[3]:= ParametricPlot[{Cos[t], Sin[t]}, {t, 0, 2 Pi}];
```




```
In[5]:= ParametricPlot3D[{Cos[t], Sin[t], t}, {t, 0, 2 Pi}];
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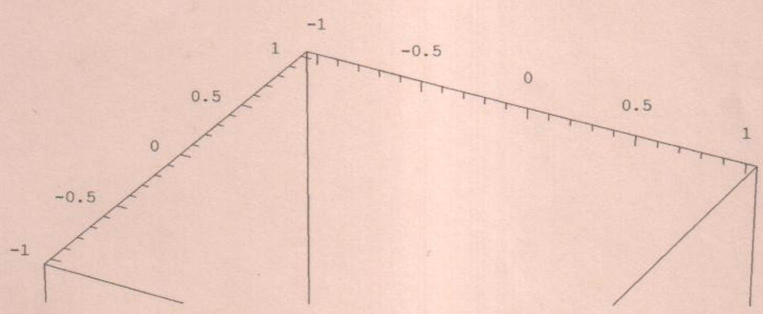

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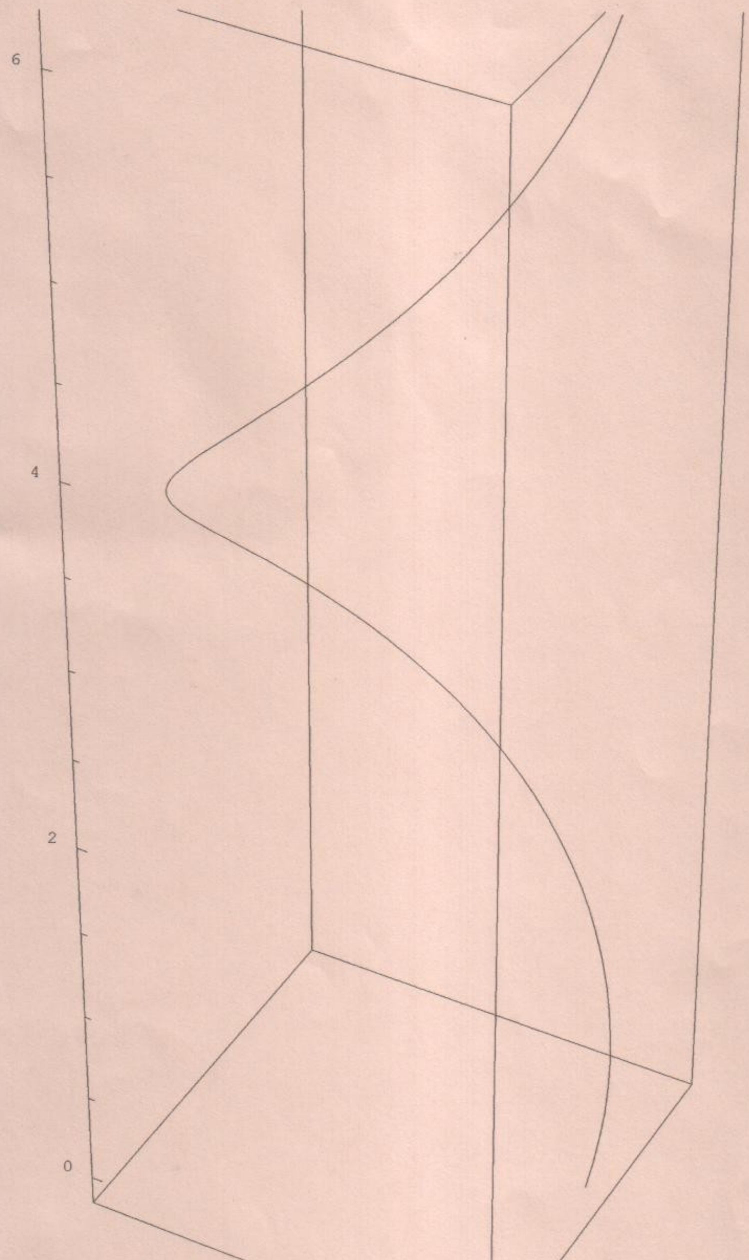

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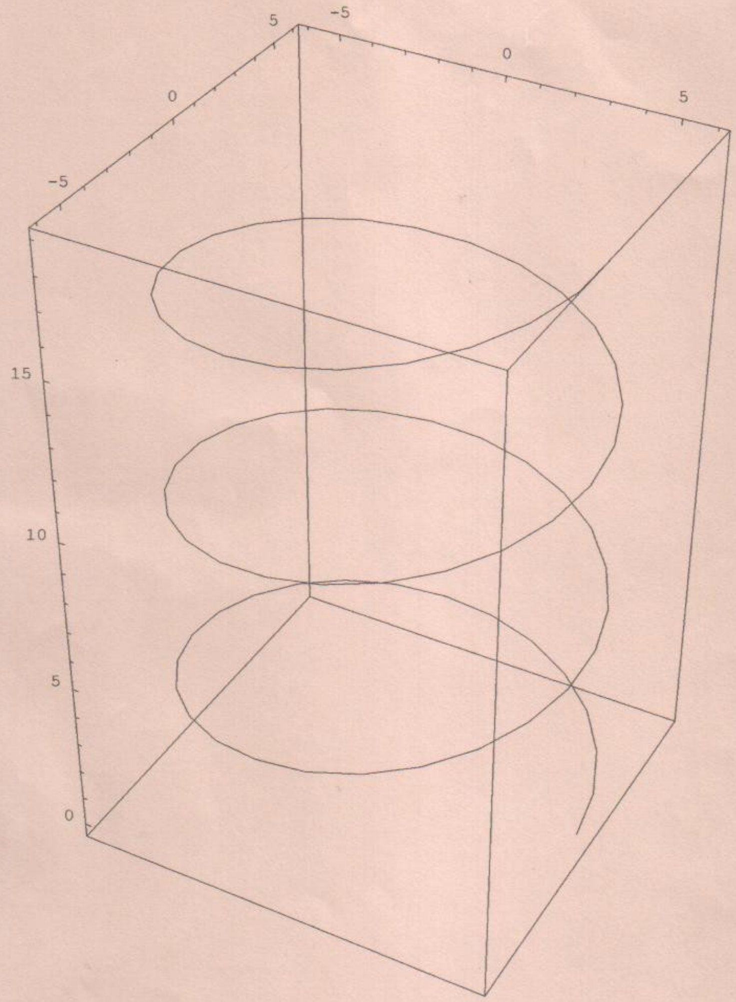
In[6]:= ParametricPlot3D[{6 Cos[t], 6 Sin[t], t}, {t, 0, 6 Pi}];
```



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Visual produced in Steven Wolfram's Mathematica with the assistance of Joshua Tepper [redacted]

```
In[10]:= ParametricPlot3D[{20 Cos[t], 20 Sin[t], t}, {t, 0, 20 Pi}, PlotPoints → 1000];
```

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